# geographiclib Documentation 

Release 1.49

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The documentation for other versions is available at https://geographiclib.sourceforge.io/m.nn/ python/for versions numbers m.nn 1.46.
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## CHAPTER 1

## Introduction

This is a python implementation of the geodesic routines in GeographicLib.
Although it is maintained in conjunction with the larger C++ library, this python package can be used independently.

### 1.1 Installation

The full Geographic package can be downloaded from sourceforge. However the python implementation is available as a stand-alone package. To install this, run
pip install geographiclib

Alternatively downloaded the package directly from Python Package Index and install it with

```
tar xpfz geographiclib-1.49.tar.gz
cd geographiclib-1.49
python setup.py install
```

It's a good idea to run the unit tests to verify that the installation worked OK by running

```
python -m unittest geographiclib.test.test_geodesic
```


### 1.2 Contents

### 1.2.1 Geodesics on an ellipsoid

Jump to

- Introduction
- Additional properties
- Multiple shortest geodesics
- Background
- References


## Introduction

Consider a ellipsoid of revolution with equatorial radius $a$, polar semi-axis $b$, and flattening $f=(a b) / a$. Points on the surface of the ellipsoid are characterized by their latitude $\phi$ and longitude $\lambda$. (Note that latitude here means the geographical latitude, the angle between the normal to the ellipsoid and the equatorial plane).
The shortest path between two points on the ellipsoid at $\left(\phi_{1}, \lambda_{1}\right)$ and $\left(\phi_{2}, \lambda_{2}\right)$ is called the geodesic. Its length is $s_{12}$ and the geodesic from point 1 to point 2 has forward azimuths $\alpha_{1}$ and $\alpha_{2}$ at the two end points. In this figure, we have $\lambda_{12}=\lambda_{2} \lambda_{1}$.

A geodesic can be extended indefinitely by requiring that any sufficiently small segment is a shortest path; geodesics are also the straightest curves on the surface.
Traditionally two geodesic problems are considered:

- the direct problem - given $\phi_{1}, \lambda_{1}, \alpha_{1}, s_{12}$, determine $\phi_{2}, \lambda_{2}$, and $\alpha_{2}$; this is solved by Geodesic.Direct.
- the inverse problem - given $\phi_{1}, \lambda_{1}, \phi_{2}, \lambda_{2}$, determine $s_{12}, \alpha_{1}$, and $\alpha_{2}$; this is solved by Geodesic. Inverse.


## Additional properties

The routines also calculate several other quantities of interest

- $S_{12}$ is the area between the geodesic from point 1 to point 2 and the equator; i.e., it is the area, measured counter-clockwise, of the quadrilateral with corners $\left(\phi_{1}, \lambda_{1}\right),\left(0, \lambda_{1}\right),\left(0, \lambda_{2}\right)$, and ( $\phi_{2}, \lambda_{2}$ ). It is given in meters ${ }^{2}$.
- $m_{12}$, the reduced length of the geodesic is defined such that if the initial azimuth is perturbed by $d \alpha_{1}$ (radians) then the second point is displaced by $m_{12} d \alpha_{1}$ in the direction perpendicular to the geodesic. $m_{12}$ is given in meters. On a curved surface the reduced length obeys a symmetry relation, $m_{12}+m_{21}=0$. On a flat surface, we have $m_{12}=s_{12}$.
- $M_{12}$ and $M_{21}$ are geodesic scales. If two geodesics are parallel at point 1 and separated by a small distance $d t$, then they are separated by a distance $M_{12} d t$ at point $2 . M_{21}$ is defined similarly (with the geodesics being parallel to one another at point 2). $M_{12}$ and $M_{21}$ are dimensionless quantities. On a flat surface, we have $M_{12}=$ $M_{21}=1$.
- $\sigma_{12}$ is the arc length on the auxiliary sphere. This is a construct for converting the problem to one in spherical trigonometry. The spherical arc length from one equator crossing to the next is always $180^{\circ}$.

If points 1,2 , and 3 lie on a single geodesic, then the following addition rules hold:

- $s_{13}=s_{12}+s_{23}$
- $\sigma_{13}=\sigma_{12}+\sigma_{23}$
- $S_{13}=S_{12}+S_{23}$
- $m_{13}=m_{12} M_{23}+m_{23} M_{21}$
- $M_{13}=M_{12} M_{23}\left(1 M_{12} M_{21}\right) m_{23} / m_{12}$
- $M_{31}=M_{32} M_{21}\left(1 \quad M_{23} M_{32}\right) m_{12} / m_{23}$


## Multiple shortest geodesics

The shortest distance found by solving the inverse problem is (obviously) uniquely defined. However, in a few special cases there are multiple azimuths which yield the same shortest distance. Here is a catalog of those cases:

- $\phi_{1}=\phi_{2}$ (with neither point at a pole). If $\alpha_{1}=\alpha_{2}$, the geodesic is unique. Otherwise there are two geodesics and the second one is obtained by setting $\left[\alpha_{1}, \alpha_{2}\right] \leftarrow\left[\alpha_{2}, \alpha_{1}\right],\left[M_{12}, M_{21}\right] \leftarrow\left[M_{21}, M_{12}\right], S_{12} \leftarrow S_{12}$. (This occurs when the longitude difference is near $\pm 180^{\circ}$ for oblate ellipsoids.)
- $\lambda_{2}=\lambda_{1} \pm 180^{\circ}$ (with neither point at a pole). If $\alpha_{1}=0^{\circ}$ or $\pm 180^{\circ}$, the geodesic is unique. Otherwise there are two geodesics and the second one is obtained by setting [ $\left.\alpha_{1}, \alpha_{2}\right] \leftarrow\left[\alpha_{1}, \alpha_{2}\right], S_{12} \leftarrow S_{12}$. (This occurs when $\phi_{2}$ is near $\phi_{1}$ for prolate ellipsoids.)
- Points 1 and 2 at opposite poles. There are infinitely many geodesics which can be generated by setting [ $\alpha_{1}, \alpha_{2}$ ] $\leftarrow\left[\alpha_{1}, \alpha_{2}\right]+[\delta, \delta]$, for arbitrary $\delta$. (For spheres, this prescription applies when points 1 and 2 are antipodal.)
- $s_{12}=0$ (coincident points). There are infinitely many geodesics which can be generated by setting [ $\alpha_{1}, \alpha_{2}$ ] $\leftarrow$ $\left[\alpha_{1}, \alpha_{2}\right]+[\delta, \delta]$, for arbitrary $\delta$.


## Background

The algorithms implemented by this package are given in Karney (2013) and are based on Bessel (1825) and Helmert (1880); the algorithm for areas is based on Danielsen (1989). These improve on the work of Vincenty (1975) in the following respects:

- The results are accurate to round-off for terrestrial ellipsoids (the error in the distance is less then 15 nanometers, compared to 0.1 mm for Vincenty).
- The solution of the inverse problem is always found. (Vincenty's method fails to converge for nearly antipodal points.)
- The routines calculate differential and integral properties of a geodesic. This allows, for example, the area of a geodesic polygon to be computed.


## References

- F. W. Bessel, The calculation of longitude and latitude from geodesic measurements (1825), Astron. Nachr. 331(8), 852-861 (2010), translated by C. F. F. Karney and R. E. Deakin.
- F. R. Helmert, Mathematical and Physical Theories of Higher Geodesy, Vol 1, (Teubner, Leipzig, 1880), Chaps. 5-7.
- T. Vincenty, Direct and inverse solutions of geodesics on the ellipsoid with application of nested equations, Survey Review 23(176), 88-93 (1975).
- J. Danielsen, The area under the geodesic, Survey Review 30(232), 61-66 (1989).
- C. F. F. Karney, Algorithms for geodesics, J. Geodesy 87(1) 43-55 (2013); addenda.
- C. F. F. Karney, Geodesics on an ellipsoid of revolution, Feb. 2011; errata.
- A geodesic bibliography.
- The wikipedia page, Geodesics on an ellipsoid.


### 1.2.2 The library interface

Jump to

- The units
- Geodesic dictionary
- outmask and caps
- Restrictions on the parameters


## The units

All angles (latitude, longitude, azimuth, arc length) are measured in degrees with latitudes increasing northwards, longitudes increasing eastwards, and azimuths measured clockwise from north. For a point at a pole, the azimuth is defined by keeping the longitude fixed, writing $\phi= \pm\left(90^{\circ} \epsilon\right)$, and taking the limit $\epsilon \rightarrow 0+$

## Geodesic dictionary

The results returned by Geodesic.Direct, Geodesic.Inverse, GeodesicLine.Position, etc., return a dictionary with some of the following 12 fields set:

- latl $=\phi_{1}$, latitude of point 1 (degrees)
- lon $1=\lambda_{1}$, longitude of point 1 (degrees)
- azil $=\alpha_{1}$, azimuth of line at point 1 (degrees)
- lat $2=\phi_{2}$, latitude of point 2 (degrees)
- lon2 $=\lambda_{2}$, longitude of point 2 (degrees)
- azi2 $=\alpha_{2}$, (forward) azimuth of line at point 2 (degrees)
- $s 12=s_{12}$, distance from 1 to 2 (meters)
- $a 12=\sigma_{12}$, arc length on auxiliary sphere from 1 to 2 (degrees)
- $m 12=m_{12}$, reduced length of geodesic (meters)
- M12 $=M_{12}$, geodesic scale at 2 relative to 1 (dimensionless)
- $M 21=M_{21}$, geodesic scale at 1 relative to 2 (dimensionless)
- $S 12=S_{12}$, area between geodesic and equator $\left(\right.$ meters $\left.^{2}\right)$


## outmask and caps

By default, the geodesic routines return the 7 basic quantities: lat1, lon 1, azil, lat2, lon 2 , azi2, sl2, together with the arc length a12. The optional output mask parameter, outmask, can be used to tailor which quantities to calculate. In addition, when a GeodesicLine is constructed it can be provided with the optional capabilities parameter, caps, which specifies what quantities can be returned from the resulting object.
Both outmask and caps are obtained by or'ing together the following values

- EMPTY, no capabilities, no output
- LATITUDE, compute latitude, lat2
- LONGITUDE, compute longitude, lon2
- AZIMUTH, compute azimuths, azil and azi2
- DISTANCE, compute distance, sl2
- StANDARD, all of the above
- DISTANCE_IN, allow s 12 to be used as input in the direct problem
- REDUCEDLENGTH, compute reduced length, m12
- GEODESICSCALE, compute geodesic scales, M12 and M21
- AREA, compute area, S12
- ALL, all of the above;
- LONG_UNROLL, unroll longitudes

DISTANCE_IN is a capability provided to the GeodesicLine constructor. It allows the position on the line to specified in terms of distance. (Without this, the position can only be specified in terms of the arc length.) This only makes sense in the caps parameter.

LONG_UNROLL controls the treatment of longitude. If it is not set then the lon1 and lon2 fields are both reduced to the range $\left[180^{\circ}, 180^{\circ}\right.$ ). If it is set, then lonl is as given in the function call and (lon2 lonl) determines how many times and in what sense the geodesic has encircled the ellipsoid. This only makes sense in the outmask parameter.
Note that al2 is always included in the result.

## Restrictions on the parameters

- Latitudes must lie in $\left[90^{\circ}, 90^{\circ}\right]$. Latitudes outside this range are replaced by NaNs .
- The distance $s l 2$ is unrestricted. This allows geodesics to wrap around the ellipsoid. Such geodesics are no longer shortest paths. However they retain the property that they are the straightest curves on the surface.
- Similarly, the spherical arc length al2 is unrestricted.
- Longitudes and azimuths are unrestricted; internally these are exactly reduced to the range $\left[180^{\circ}, 180^{\circ}\right.$ ); but see also the LONG_UNROLL bit.
- The equatorial radius $a$ and the polar semi-axis $b$ must both be positive and finite (this implies that $\infty<f<1$ ).
- The flattening $f$ should satisfy $f[1 / 50,1 / 50]$ in order to retain full accuracy. This condition holds for most applications in geodesy.
Reasonably accurate results can be obtained for $0.2 f 0.2$. Here is a table of the approximate maximum error (expressed as a distance) for an ellipsoid with the same equatorial radius as the WGS84 ellipsoid and different values of the flattening.

| abs $(f)$ | error |
| :--- | :--- |
| 0.003 | 15 nm |
| 0.01 | 25 nm |
| 0.02 | 30 nm |
| 0.05 | $10 \mu \mathrm{~m}$ |
| 0.1 | 1.5 mm |
| 0.2 | 300 mm |

Here $1 \mathrm{~nm}=1$ nanometer $=10^{9} \mathrm{~m}$ (not 1 nautical mile!)

### 1.2.3 GeographicLib API

## geographiclib

geographiclib: geodesic routines from GeographicLib
geographiclib.__version_info__ $=(1,49,0)$
GeographicLib version as a tuple
geographiclib.__version__ = '1.49'
GeographicLib version as a string

## geographiclib.geodesic

Define the Geodesic class
The ellipsoid parameters are defined by the constructor. The direct and inverse geodesic problems are solved by

- Inverse () Solve the inverse geodesic problem
- Direct () Solve the direct geodesic problem
- ArcDirect () Solve the direct geodesic problem in terms of spherical arc length

GeodesicLine objects can be created with

- Line()
- DirectLine()
- ArcDirectLine()
- InverseLine()

PolygonArea objects can be created with

- Polygon()

The public attributes for this class are

- af
outmask and caps bit masks are
- EMPTY
- LATITUDE
- LONGITUDE
- AZIMUTH
- DISTANCE
- STANDARD
- DISTANCE_IN
- REDUCEDLENGTH
- GEODESICSCALE
- AREA
- ALL
- LONG_UNROLL


## Example

```
>>> from geographiclib.geodesic import Geodesic
>>> # The geodesic inverse problem
...Geodesic.WGS84.Inverse(-41.32, 174.81, 40.96, -5.50)
{'lat1': -41.32,
'a12': 179.6197069334283,
's12': 19959679.26735382,
'lat2': 40.96,
'azi2': 18.825195123248392,
'azi1': 161.06766998615882,
'lon1': 174.81,
'lon2': -5.5}
```

Geodesic.WGS84 = Instantiation for the WGS84 ellipsoid
class geographiclib.geodesic.Geodesic $(a, f)$

Solve geodesic problems
Construct a Geodesic object

## Parameters

- a - the equatorial radius of the ellipsoid in meters
- $\mathbf{f}$ - the flattening of the ellipsoid

An exception is thrown if $a$ or the polar semi-axis $b=a(1-f)$ is not a finite positive quantity.
a = None
The equatorial radius in meters (readonly)
f = None
The flattening (readonly)
Inverse (lat1, lon1, lat2, lon2, outmask=1929)
Solve the inverse geodesic problem

## Parameters

- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- lat2 - latitude of the second point in degrees
- lon2 - longitude of the second point in degrees
- outmask - the output mask

Returns a Geodesic dictionary
Compute geodesic between (lat1, lon1) and (lat2, lon2). The default value of outmask is STANDARD, i.e., the lat1, lon1, azil, lat2, lon2, azi2, s12, al2 entries are returned.

Direct (lat1, lon1, azil, s12, outmask=1929)
Solve the direct geodesic problem

## Parameters

- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- azi1 - azimuth at the first point in degrees
- s12 - the distance from the first point to the second in meters
- outmask - the output mask

Returns a Geodesic dictionary
Compute geodesic starting at (latl, lonl) with azimuth azil and length sl2. The default value of outmask is STANDARD, i.e., the lat1, lon1, azil, lat2, lon2, azi2, sl2, al2 entries are returned.
ArcDirect (lat1, lon1, azil, al2, outmask=1929)
Solve the direct geodesic problem in terms of spherical arc length

## Parameters

- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- azi1 - azimuth at the first point in degrees
- a12 - spherical arc length from the first point to the second in degrees
- outmask - the output mask

Returns a Geodesic dictionary
Compute geodesic starting at (lat1, lonl) with azimuth azil and arc length al2. The default value of outmask is STANDARD, i.e., the lat1, lon1, azil, lat2, lon2, azi2, s12, al2 entries are returned.
Line (lat1, lon1, azil, caps=3979)
Return a GeodesicLine object

## Parameters

- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- azi1 - azimuth at the first point in degrees
- caps - the capabilities


## Returns a GeodesicLine

This allows points along a geodesic starting at (lat1, lon1), with azimuth azil to be found. The default value of caps is STANDARD I DISTANCE_IN, allowing direct geodesic problem to be solved.
DirectLine (lat1, lon1, azil, s12, caps=3979)
Define a GeodesicLine object in terms of the direct geodesic problem specified in terms of spherical arc length

## Parameters

- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- azi1 - azimuth at the first point in degrees
- s12 - the distance from the first point to the second in meters
- caps - the capabilities

Returns a GeodesicLine
This function sets point 3 of the GeodesicLine to correspond to point 2 of the direct geodesic problem. The default value of caps is STANDARD I DISTANCE_IN, allowing direct geodesic problem to be solved.

ArcDirectLine (lat1, lon1, azil, a12, caps=3979)
Define a GeodesicLine object in terms of the direct geodesic problem specified in terms of spherical arc length

## Parameters

- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- azi1 - azimuth at the first point in degrees
- a12 - spherical arc length from the first point to the second in degrees
- caps - the capabilities

Returns a GeodesicLine
This function sets point 3 of the GeodesicLine to correspond to point 2 of the direct geodesic problem. The default value of caps is STANDARD I DISTANCE_IN, allowing direct geodesic problem to be solved.
InverseLine (lat1, lon1, lat2, lon2, caps=3979)
Define a GeodesicLine object in terms of the invese geodesic problem

## Parameters

- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- lat2 - latitude of the second point in degrees
- lon2 - longitude of the second point in degrees
- caps - the capabilities

Returns a GeodesicLine
This function sets point 3 of the GeodesicLine to correspond to point 2 of the inverse geodesic problem. The default value of caps is STANDARD I DISTANCE_IN, allowing direct geodesic problem to be solved.

```
Polygon (polyline=False)
```

Return a PolygonArea object
Parameters polyline - if True then the object describes a polyline instead of a polygon
Returns a PolygonArea

## EMPTY $=0$

No capabilities, no output.
LATITUDE $=128$
Calculate latitude lat2.
LONGITUDE = 264
Calculate longitude lon2.
AZIMUTH $=512$
Calculate azimuths azil and azi2.
DISTANCE $=1025$
Calculate distance s12.
STANDARD $=1929$
All of the above.

DISTANCE_IN = 2051
Allow distance sl2 to be used as input in the direct geodesic problem.
REDUCEDLENGTH $=4101$
Calculate reduced length $m 12$.
GEODESICSCALE $=8197$
Calculate geodesic scales M12 and M21.
AREA $=16400$
Calculate area S12.
ALL = 32671
All of the above.
LONG_UNROLL = 32768
Unroll longitudes, rather than reducing them to the range [-180d,180d].

## geographiclib.geodesicline

Define the GeodesicLine class
The constructor defines the starting point of the line. Points on the line are given by

- Position () position given in terms of distance
- ArcPosition () position given in terms of spherical arc length

A reference point 3 can be defined with

- SetDistance () set position of 3 in terms of the distance from the starting point
- SetArc () set position of 3 in terms of the spherical arc length from the starting point

The object can also be constructed by

- Geodesic.Line
- Geodesic.DirectLine
- Geodesic.ArcDirectLine
- Geodesic.InverseLine

The public attributes for this class are

- af caps lat1 lonl azil salpl calpl sl3a13
class geographiclib.geodesicline.GeodesicLine(geod, latl, lonl, azil, caps=3979, salp1=nan, calp1=nan)
Points on a geodesic path
Construct a GeodesicLine object


## Parameters

- geod-a Geodesic object
- lat1 - latitude of the first point in degrees
- lon1 - longitude of the first point in degrees
- azi1 - azimuth at the first point in degrees
- caps - the capabilities

This creates an object allowing points along a geodesic starting at (latl, lonl), with azimuth azil to be found. The default value of caps is STANDARD I DISTANCE_IN. The optional parameters salp1 and calp1 should not be supplied; they are part of the private interface.

```
a = None
The equatorial radius in meters (readonly)
```

```
f = None
    The flattening (readonly)
```


## caps = None

the capabilities (readonly)
lat1 = None
the latitude of the first point in degrees (readonly)

## lon1 = None

the longitude of the first point in degrees (readonly)
azil = None
the azimuth at the first point in degrees (readonly)

## salp1 = None

the sine of the azimuth at the first point (readonly)

```
calp1 = None
```

the cosine of the azimuth at the first point (readonly)

## s13 = None

the distance between point 1 and point 3 in meters (readonly)

## a13 = None

the arc length between point 1 and point 3 in degrees (readonly)
Position (s12, outmask=1929)
Find the position on the line given $s 12$

## Parameters

- s12 - the distance from the first point to the second in meters
- outmask - the output mask

Returns a Geodesic dictionary
The default value of outmask is STANDARD, i.e., the lat1, lon1, azil, lat2, lon2, azi2, s12, al2 entries are returned. The GeodesicLine object must have been constructed with the DISTANCE_IN capability.
ArcPosition (a12, outmask=1929)
Find the position on the line given al2

## Parameters

- a12 - spherical arc length from the first point to the second in degrees
- outmask - the output mask


## Returns a Geodesic dictionary

The default value of outmask is STANDARD, i.e., the lat1, lon1, azi1, lat2, lon2, azi2, s12, al2 entries are returned.
SetDistance (sl3)
Specify the position of point 3 in terms of distance
Parameters s13-distance from point 1 to point 3 in meters

SetArc (al3)
Specify the position of point 3 in terms of arc length
Parameters a13 - spherical arc length from point 1 to point 3 in degrees

## geographiclib.polygonarea

Define the PolygonArea class
The constructor initializes a empty polygon. The available methods are

- Clear() reset the polygon
- AddPoint () add a vertex to the polygon
- AddEdge () add an edge to the polygon
- Compute () compute the properties of the polygon
- TestPoint () compute the properties of the polygon with a tentative additional vertex
- TestEdge () compute the properties of the polygon with a tentative additional edge

The public attributes for this class are

- earth polyline area 0 num latl lonl
class geographiclib.polygonarea.PolygonArea(earth, polyline=False)
Area of a geodesic polygon
Construct a PolygonArea object


## Parameters

- earth - a Geodesic object
- polyline - if true, treat object as a polyline instead of a polygon

Initially the polygon has no vertices.

```
earth = None
```

The geodesic object (readonly)
polyline $=$ None
Is this a polyline? (readonly)
area0 $=$ None
The total area of the ellipsoid in meter^2 (readonly)
num $=$ None
The current number of points in the polygon (readonly)
lat1 $=$ None
The current latitude in degrees (readonly)
lon1 = None
The current longitude in degrees (readonly)
Clear ()
Reset to empty polygon.
AddPoint (lat, lon)
Add the next vertex to the polygon

## Parameters

- lat - the latitude of the point in degrees
- Ion - the longitude of the point in degrees

This adds an edge from the current vertex to the new vertex.
AddEdge (azi, $s$ )
Add the next edge to the polygon

## Parameters

- azi - the azimuth at the current the point in degrees
- $\boldsymbol{s}$ - the length of the edge in meters

This specifies the new vertex in terms of the edge from the current vertex.
Compute $($ reverse $=$ False, sign=True )
Compute the properties of the polygon

## Parameters

- reverse - if true then clockwise (instead of counter-clockwise) traversal counts as a positive area
- sign - if true then return a signed result for the area if the polygon is traversed in the "wrong" direction instead of returning the area for the rest of the earth

Returns a tuple of number, perimeter (meters), area (meters ${ }^{\wedge} 2$ )
If the object is a polygon (and not a polygon), the perimeter includes the length of a final edge connecting the current point to the initial point. If the object is a polyline, then area is nan.
More points can be added to the polygon after this call.
TestPoint (lat, lon, reverse=False, sign=True)
Compute the properties for a tentative additional vertex

## Parameters

- lat - the latitude of the point in degrees
- Ion - the longitude of the point in degrees
- reverse - if true then clockwise (instead of counter-clockwise) traversal counts as a positive area
- sign - if true then return a signed result for the area if the polygon is traversed in the "wrong" direction instead of returning the area for the rest of the earth
Returns a tuple of number, perimeter (meters), area (meters^2)
TestEdge (azi, s, reverse $=$ False, sign=True )
Compute the properties for a tentative additional edge


## Parameters

- azi - the azimuth at the current the point in degrees
- s-the length of the edge in meters
- reverse - if true then clockwise (instead of counter-clockwise) traversal counts as a positive area
- sign - if true then return a signed result for the area if the polygon is traversed in the "wrong" direction instead of returning the area for the rest of the earth


## Returns a tuple of number, perimeter (meters), area (meters^2)

## geographiclib.constants

Define the WGS84 ellipsoid

```
class geographiclib.constants.Constants
```

    Constants describing the WGS84 ellipsoid
    WGS84_a = 6378137.0
        the equatorial radius in meters of the WGS84 ellipsoid in meters
        WGS84_f = 0.0033528106647474805
        the flattening of the WGS84 ellipsoid, 1/298.257223563
    
### 1.2.4 Examples

## Initializing

The following examples all assume that the following commands have been carried out:

```
>>> from geographiclib.geodesic import Geodesic
>>> import math
>>> geod = Geodesic.WGS84 # define the WGS84 ellipsoid
```

You can determine the ellipsoid parameters with the $a$ and $f$ member variables, for example,

```
>>> geod.a, 1/geod.f
(6378137.0, 298.257223563)
```

If you need to use a different ellipsoid, construct one by, for example

```
>>> geod = Geodesic(6378388, 1/297.0) # the international ellipsoid
```


## Basic geodesic calculations

The distance from Wellington, NZ (41.32S, 174.81E) to Salamanca, Spain (40.96N, 5.50W) using Inverse ():

```
>>> g = geod.Inverse(-41.32, 174.81, 40.96, -5.50)
>>> print "The distance is {:.3f} m.".format(g['s12'])
The distance is 19959679.267 m.
```

The point the point 20000 km SW of Perth, Australia (32.06S, 115.74E) using Direct ():

```
>>> g = geod.Direct(-32.06, 115.74, 225, 20000e3)
>>> print "The position is ({:.8f}, {:.8f}).".format(g['lat2'],g['lon2'])
The position is (32.11195529, -63.95925278).
```

The area between the geodesic from JFK Airport ( $40.6 \mathrm{~N}, 73.8 \mathrm{~W}$ ) to LHR Airport ( $51.6 \mathrm{~N}, 0.5 \mathrm{~W}$ ) and the equator. This is an example of setting the the output mask parameter.

```
>>> g = geod.Inverse(40.6, -73.8, 51.6, -0.5, Geodesic.AREA)
>>> print "The area is {:.If} m^2".format(g['S12'])
The area is 40041368848742.5 m^2
```


## Computing waypoints

Consider the geodesic between Beijing Airport (40.1N, 116.6E) and San Fransisco Airport (37.6N, 122.4W). Compute waypoints and azimuths at intervals of 1000 km using Geodesic. Line and GeodesicLine.Position:

```
>>> l = geod.InverseLine(40.1, 116.6, 37.6, -122.4)
>>> ds = 1000e3; n = int(math.ceil(l.s13 / ds))
>>> for i in range(n + 1):
... if i == 0:
... print "distance latitude longitude azimuth"
... s = min(ds * i, l.sl3)
... g = l.Position(s, Geodesic.STANDARD | Geodesic.LONG_UNROLL)
... print "{:.Of} {:.5f} {:.5f} {:.5f}".format(
... g['s12'], g['lat2'], g['lon2'], g['azi2'])
. . .
distance latitude longitude azimuth
0 40.10000 116.60000 42.91642
1000000 46.37321 125.44903 48.99365
2000000 51.78786 136.40751 57.29433
3000000 55.92437 149.93825 68.24573
4000000 58.27452 165.90776 81.68242
5000000 58.43499 183.03167 96.29014
6000000 56.37430 199.26948 109.99924
7000000 52.45769 213.17327 121.33210
8000000 47.19436 224.47209 129.98619
9000000 41.02145 233.58294 136.34359
9513998 37.60000 237.60000 138.89027
```

The inclusion of Geodesic.LONG_UNROLL in the call to GeodesicLine.Position ensures that the longitude does not jump on crossing the international dateline.

If the purpose of computing the waypoints is to plot a smooth geodesic, then it's not important that they be exactly equally spaced. In this case, it's faster to parameterize the line in terms of the spherical arc length with GeodesicLine.ArcPosition. Here the spacing is about $1^{\circ}$ of arc which means that the distance between the waypoints will be about 60 NM .

```
>>> l = geod.InverseLine(40.1, 116.6, 37.6, -122.4,
... Geodesic.LATITUDE | Geodesic.LONGITUDE)
>>> da = 1; n = int(math.ceil(l.a13 / da)); da = l.a13 / n
>>> for i in range(n + 1):
... if i == 0:
... print "latitude longitude"
... a = da * i
... g = l.ArcPosition(a, Geodesic.LATITUDE |
... Geodesic.LONGITUDE | Geodesic.LONG_UNROLL)
... print "{:.5f} {:.5f}".format(g['lat2'], g['lon2'])
...
latitude longitude
40.10000 116.60000
40.82573 117.49243
41.54435 118.40447
42.25551 119.33686
42.95886 120.29036
43.65403 121.26575
44.34062 122.26380
...
39.82385 235.05331
39.08884 235.91990
```

```
38.34746 236.76857
37.60000 237.60000
```

The variation in the distance between these waypoints is on the order of $1 / f$.

## Measuring areas

Measure the area of Antarctica using Geodesic.Polygon and the PolygonArea class:

```
>>> p = geod.Polygon()
>>> antarctica = [
... [-63.1, -58], [-72.9, -74], [-71.9,-102], [-74.9,-102], [-74.3,-131],
... [-77.5,-163], [-77.4, 163], [-71.7, 172], [-65.9, 140], [-65.7, 113],
... [-66.6, 88], [-66.9, 59], [-69.8, 25], [-70.0, -4], [-71.0, -14],
... [-77.3, -33], [-77.9, -46], [-74.7, -61]
... ]
>>> for pnt in antarctica:
... p.AddPoint(pnt[0], pnt[1])
•••
>>> num, perim, area = p.Compute()
>>> print "Perimeter/area of Antarctica are {:.3f} m / {:.If} m^2".format(
    perim, area)
Perimeter/area of Antarctica are 16831067.893 m / 13662703680020.1 m^2
```


### 1.3 GeographicLib in various languages

- $\mathrm{C}++$ (complete library): documentation, download
- C (geodesic routines): documentation, also included with recent versions of proj. 4
- Fortran (geodesic routines): documentation
- Java (geodesic routines): Maven Central package, documentation
- JavaScript (geodesic routines): npm package, documentation
- Python (geodesic routines): PyPI package, documentation
- Matlab/Octave (geodesic and some other routines): Matlab Central package, documentation
- C\# (.NET wrapper for complete C++ library): documentation


### 1.4 Change log

- Version 1.49 (released 2017-10-05)
- Fix code formatting; add tests.
- Version 1.48 (released 2017-04-09)
- Change default range for longitude and azimuth to $\left(180^{\circ}, 180^{\circ}\right]$ (instead of $\left[180^{\circ}, 180^{\circ}\right)$ ).
- Version 1.47 (released 2017-02-15)
- Fix the packaging, incorporating the patches in version 1.46.3.
- Improve accuracy of area calculation (fixing a flaw introduced in version 1.46)
- Version 1.46 (released 2016-02-15)
- Add Geodesic.DirectLine, Geodesic.ArcDirectLine, Geodesic.InverseLine, GeodesicLine.SetDistance, GeodesicLine.SetArc, GeodesicLine.s13, GeodesicLine.a13.
- More accurate inverse solution when longitude difference is close to $180^{\circ}$.
- Remove unnecessary functions, CheckPosition, CheckAzimuth, CheckDistance.


## CHAPTER 2

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